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An Algorithm for Solving Fully Fuzzy Rough Integer Transportation problems

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Abstract:

A suggested algorithm, for solving fully fuzzy rough integer transportation problems (FFRITPs), is introduced in order to find an optimal solution and optimal values cost where the supply and demand are balanced. In real-life situations, the parameters of a transportation problems model may not be defined precisely, because of the current market globalization and some other uncontrollable factors. By study a special cases when all parameters and decision variables in the constraints and the objective function are trapezoidal fuzzy rough number (TrFRN). The methodology of the solution depends on dividing this problem into six crisp problems, and then assemble the solutions to find the optimal solution for the original problem. Furthermore, the proposed algorithm enable us to search for the optimal solution in the largest range of possible solutions. The split and separation method can be served as an important tool for the decision makers when they are handling various types of logistic problems having trapezoidal fuzzy rough variable and parameters. [In addition, the motivation behind this study is to enable the decision makers to make the right decision considering the proposed solutions, while dealing with the uncertain and imprecise data] Finally, the effectiveness of the proposed procedure is illustrated through numerical example.

Keywords: Fuzzy number- Fuzzy rough number- Transportation problem- fully fuzzy rough problem- Optimal solution- supply and demand.

خوارزمية لحل مشاكل النقل الإستقرابية الفازية الصحيحة

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الملخص:

في مشاكل البرمجة الخطية وكذا مشاكل النقل عادة ما تواجهنا في الحياة العملية حالة من عدم اليقين نتيجة لعدم القدرة على تحديد معاملات النموذج وكذلك متغيرات القرار بشكل دقيق و بسبب العديد من العوامل التي لا يمكن السيطرة عليها.

اقترحنا في هدا البحث خوارزمية لحل مشاكل النقل وإيجاد الحل الأمثل وتكلفة القيم المثلى حيث يكون العرض والطلب متوازنين عندما تكون البيانات غير يقينية، وذلك من خلال دراسة مشاكل النقل التي تكون فيها متغيرات القرار والمعاملات في دالة الهدف والقيود عبارة عن (trapezoidal fuzzy rough number).

تعتمد منهجية الحل إلى تقسيم المشكلة إلى ست مشاكل ذات أعداد ثابتة ثم يتم تجميع الحل للمشكلة للحصول على الحل الأمثل لمثل هذه المشاكل التي يسودها عدم اليقين.

علاوة على ذلك تمكننا الخوارزمية من البحث عن الحل الامثل في أكبر عدد من الحلول الممكنة وبالتالي تمكين صناع القرار من اتخاذ القرار الصحيح بالنظر إلى الحلول المقترحة. أخيرا تم تقديم بعض الأمثلة لتوضيح طريقة عمل الخوارزمية.

الكلمات المفتاحية: العدد الفازي- العدد الإستقرابي الفازي- مشكلة النقل- المشكلة الإستقرابية الفازية الكاملة- الحل الأمثل- العرض والطلب.

1. Introduction

Transportation problem (TP) is one of the popular and most important applications of the linear programming problem. Many efficient algorithms have been developed for solving TPs having deterministic parameters. In many real-life situations, some or all parameters of the TP are not deterministic always, but they are uncertain. (Zimmermann, 1978) developed Zimmermanns fuzzy linear programming into several fuzzy optimization methods for solving TPs. Many researchers (Ebrahimnejad, 2015), (Kaur and Kumar, 2012), (Natarajan, 2010) and (Shanmugasundari and Ganesan, 2013) have proposed various methods to solve interval and fuzzy TPs. The theory of rough sets proposed by (Pawlak, 1982) which deals with approximation of an arbitrary subset of universe by two definable or observable subset called lower and upper approximation. Then, many researchers have developed the rough set theory both in theoretical and applied. A rough programming problem considering the decision set as a rough set was introduced and solved by (Youness, 2006). (Shaocheng, 1994) introduced two kinds of linear programming with fuzzy numbers. They are called interval number and fuzzy number linear programming. (Taha, 1997) Integer programming (IP) problems are optimization problems that minimize or maximize the objective function, taking into consideration the limits of constraints and integer variables. More widely, the applications of integer programming can be used to appropriately describe the decision problems concerning the effective use of resources in engineering technology, business management and other numerous fields. (Pamucar et al. 2019) proposed a new approach for the treatment of uncertainty and imprecision based on interval-valued fuzzy-rough numbers. The concept of rough variable, which is a measurable function from rough space to the set of real numbers, was proposed by (Liu, 2012). (Roy et al. 2019) presented investigate for a multi-objective multi-item fixed-charge solid transportation problem with fuzzy-rough variables as coefficients of the objective functions and of the constraints. (Pandian et al. 2016) believed that transportation problem has all or some parameters as rough integer intervals. They also proposed a new method named, a slice-sum method to solve Rough Integer Interval Transportation Problem (RIITP), where transportation cost, supply and demand are rough integer intervals. (Osman et al. 2016) presented a solution approach for rough interval multi objective transportation problem (RIMOTP). The concept of solving conventional interval programming combined with fuzzy programming is used to build the solution approach for RIMOTP. (Liu and Yuan 2007) developed triangular intuitionistic fuzzy sets based on the combination of triangular fuzzy numbers and intuitionistic fuzzy sets. (Garg et al. 2014) presented a methodology for solving the multi-objective reliability optimization model, in which parameters are considered imprecise in terms of triangular interval data. The most relevant method for solving transportation problems with interval coefficients was proposed by (Ammar & Khalifa, 2014) They relied on transformed the original intervals coefficients to crisp coefficients. (Garai et al. 2019) presented investigated a multi-objective inventory model under both stock-dependent demand rate and holding cost rate with fuzzy random coefficients. This study focuses on the development of a convenient method to solving integer transportation problems and find an optimal solution and optimal values cost where the supply and demand are balanced , where all parameters and decision variables in the constraints and the objective function are (RIs) and (Tr.F.R. N). In this work, we present a new method namely, slides solution method to

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find an optimal solution for integer TPs where transportation cost, supply and demand are fuzzy integer intervals.

This paper is organized as follows: In Section 2 some basic definitions and some arithmetic results are presented. In Section 3, formulation of trapezoidal fuzzy rough number problem and application for solving trapezoidal fuzzy rough number problem are established.
In section 4, advantages of the proposed method are discussed.. Finally, the conclusion part is present in Section 5.

2. Preliminaries

The following are some definitions of the basic arithmetic operators and partial ordering relations on rough intervals and trapezoidal fuzzy rough numbers based on the function principle which used in Section 3.

2.1. Rough Intervals

***Definition* 1.** (Rough Interval**);** Let X denoted a compact set of real numbers. A rough interval X^R is defined as an interval with known lower and upper bounds but unknown distribution information for X denoted by:
$X^R = \left[X^{(LAI)}: X^{(UAI)}\right]$ where $X^{(LAI)}$ *and* $X^{(UAI)}$ are lower and upper approximation intervals of X^R, respectively with $X^{(LAI)} \subseteq X^{(UAI)}$.

***Definition* 2.** "The arithmetic operations on RIs are depending on interval arithmetic, so we will state some of these arithmetic operations as follows:

Let $A^R = \left[[\, a^{LL}, a^{UL}]: [a^{LU}, a^{UU}]\right]$ and $B^R = \left[[\, b^{LL}, b^{UL}]: [b^{LU}, b^{UU}]\right]$ be two RIs if $A^R, B^R \geq 0$. Then.

[Addition:]
$$A^R + B^R = ([a^{LL} + b^{LL}, a^{UL} + b^{UL}]: [a^{LU} + b^{LU}, a^{UU} + b^{UU}]).$$
[Subtraction:]
$$A^R - B^R = ([a^{LL} - b^{UL}, a^{UL} - b^{LL}]: [a^{LU} - b^{UU}, a^{UU} - b^{LU}]).$$
[Multiply:]
$$A^R * B^R = ([a^{LL}b^{LL}, a^{UL}b^{UL}]: [a^{LU}b^{LU}, a^{UU}b^{UU}]).$$
[Negation:]
$$-A^R = ([-a^{\mathrm{UL}}, -a^{\mathrm{LL}}] : [-a^{\mathrm{UU}}, -a^{\mathrm{LU}}]) .$$
[Intersection:]
$$\mathrm{A^R \cap B^R} =$$

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$([max\{a^{LL}, b^{LL}\}, min\{a^{UL}, b^{UL}\}], [max\{a^{LU}, b^{LU}\}, min\{a^{UU}, b^{UU}\}])$.

[Union:]

$A^R \cup B^R =$
$([min\{a^{LL}, b^{LL}\}, max\{a^{UL}, b^{UL}\}], [min\{a^{LU}, b^{LU}\}, max\{a^{UU}, b^{UU}\}])$.

2.2. Trapezoidal Fuzzy Number

***Definition* 3.** A trapezoidal fuzzy number (Tr.F.N),A can be represented completely by a quadruplet $\tilde{A} = (a_1, a_2, a_3, a_4)$ Also, a Tr.F.N can be characterized by the interval of confidence at level α: thus,

$$A_\alpha = [a_1^\alpha, a_4^\alpha] = [(a_2 - a_1)\alpha + a_1, -(a_4 - a_3)\alpha + a_4] \quad \forall\, \alpha \in (0,1].$$

The membership function of (Tr.F.N) $\tilde{A} = (a_1, a_2, a_3, a_4)$ is characterized as:

$$\mu_{\tilde{A}}(x) \begin{cases} 0 & , \quad x < a_1 \\ \dfrac{x - a_1}{a_2 - a_1} & , \quad a_1 \le x \le a_2 \\ 1 & , \quad a_2 \le x \le a_3 \\ \dfrac{a_4 - x}{a_4 - a_3} & , \quad a_3 \le x \le a_4 \\ 0 & , \quad x \ge a_4 \end{cases}$$

Definition 4. A trapezoidal fuzzy rough number $\tilde{A}^R$ denoted by $\tilde{A}^R = [(a^{LL}, a^M, a^N, a^{UL}) : (a^{LU}, a^M, a^N, a^{UU})]$ where $a^{LU}, a^{LL}, a^M, a^N, a^{UL} and\ \ a^{UU} \in \mathcal{R}$ real number such that $a^{LU} \le a^{LL} \le a^M \le a^N \le a^{UL} \le a^{UU}$ (see Fig. 2) and the membership function can be defined as:

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$$\mu_{\tilde{A}^R}(x)\begin{cases}\mu_{\tilde{a}^L}(x)\begin{cases}0 & , \quad x < a^{LL}\\ \dfrac{x-a^{LL}}{a^M-a^{LL}} & , \quad a^{LL} \le x \le a^M\\ 1 & , \quad a^M \le x \le a^N\\ \dfrac{a^{LL}-x}{a^{UL}-a^N} & , \quad a_3 \le x \le a_4\\ 0 & , \quad x > a^{UL}\end{cases}\\ \mu_{\tilde{a}^U}(x)\begin{cases}0 & , \quad x < a^{LU}\\ \dfrac{x-a^{LU}}{a^M-a^{LU}} & , \quad a^{LU} \le x \le a^M\\ 1 & , \quad a^M \le x \le a^N\\ \dfrac{a^{UU}-x}{a^{UU}-a^N} & , \quad a^N \le x \le a^{UU}\\ 0 & , \quad otherwise\end{cases}\end{cases}$$

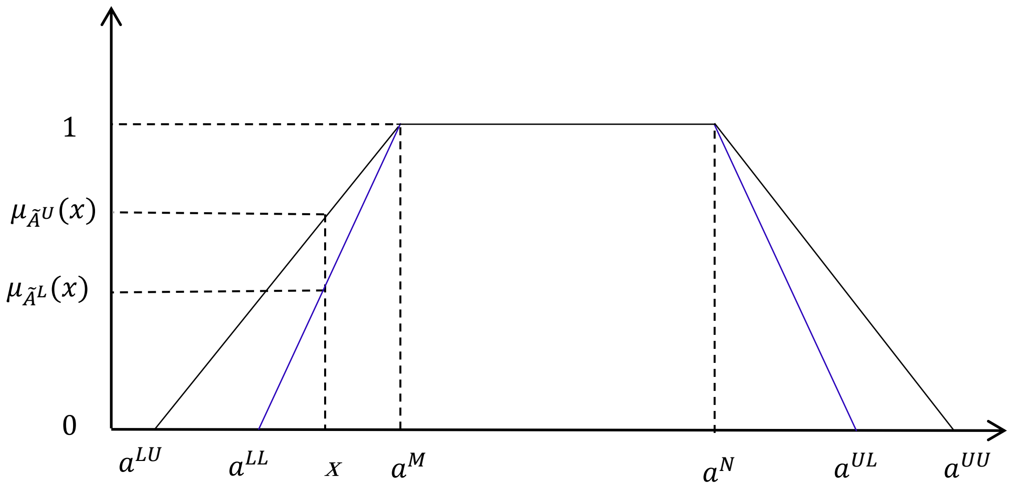

Fig. 1 Membership function of the TrFRN

Note that $\mu_{\tilde{A}^R}(x) = \left[\tilde{A}^L = (a^{LL}, a^M, a^N, a^{UL}),\ \tilde{A}^U = (a^{LU}, a^M, a^N, a^{UU})\right]$ *and* $\tilde{A}^L \subseteq \tilde{A}^U$ where $\mu_{\tilde{A}^L}(x)$ And $\mu_{\tilde{A}^U}(x)$ membership function of lower and upper approximation trapezoidal fuzzy rough number of $\mu_{\tilde{A}^R}(x)$.

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2.3. The arithmetic operations for trapezoidal Fuzzy Rough Number

$$\text{Let } \tilde{A}^R = \left[[a^{LL}, a^M, a^N, a^{UL}]: [a^{LU}, a^M, a^N, a^{UU}]\right] \text{ and}$$
$$\tilde{B}^R = \left[[b^{LL}, b^M, b^N, b^{UL}]: [b^{LU}, b^M, b^N, b^{UU}]\right]$$

be two sets trapezoidal fuzzy rough numbers, where $\tilde{A}^R$ *and* $\tilde{B}^R \geq 0$, then the arithmetic operations are defined by:

1. Addition
$$\tilde{A}^R + \tilde{B}^R = \left[(\tilde{A}^L + \tilde{B}^L): (\tilde{A}^U + \tilde{B}^U)\right] \text{ where}$$
$$\tilde{A}^L + \tilde{B}^L = (a^{LL} + b^{LL}, a^M + b^M, a^N + b^N, a^{UL} + b^{UL})$$
$$\tilde{A}^U + \tilde{B}^U = (a^{LU} + b^{LU}, a^M + b^M, a^N + b^N, a^{UU} + b^{UU})$$
2. Subtraction:
$$\tilde{A}^R - \tilde{B}^R = \left[(\tilde{A}^L - \tilde{B}^L): (\tilde{A}^U - \tilde{B}^U)\right] \text{ where}$$
$$\tilde{A}^L - \tilde{B}^L = (a^{LL} - b^{UL}, a^M - b^M, a^N - b^N, a^{UL} - b^{LL})$$
$$\tilde{A}^U - \tilde{B}^U = (a^{LU} - b^{UU}, a^M - b^M, a^N - b^N, a^{UU} - b^{LU})$$
3. Multiplication:
$$\tilde{A}^R \times \tilde{B}^R = \left[(\tilde{A}^L \times \tilde{B}^L): (\tilde{A}^U \times \tilde{B}^U)\right] \text{ where}$$
$$\tilde{A}^L \times \tilde{B}^L = (a^{LL} \times b^{LL}, a^M \times b^M, a^N \times b^N, a^{UL} \times b^{UL})$$
$$\tilde{A}^U \times \tilde{B}^U = (a^{LU} \times b^{LU}, a^M \times b^M, a^N \times b^N, a^{UU} \times b^{UU})$$
4. Division
$$\text{If } 0 \notin \tilde{B}^R \text{ then } \tilde{A}^R \div \tilde{B}^R = \left[(\tilde{A}^L \div \tilde{B}^L): (\tilde{A}^U \div \tilde{B}^U)\right] \text{ where}$$
$$\tilde{A}^L \div \tilde{B}^L = (a^{LL} \div b^{UL}, a^M \div b^M, a^N \div b^N, a^{UL} \div b^{LL})$$
$$\tilde{A}^U \div \tilde{B}^U = (a^{LU} \div b^{UU}, a^M \div b^M, a^N \div b^N, a^{UU} \div b^{LU})$$

Definition 5. *Let* $\tilde{A}^R = [\tilde{A}^L : \tilde{A}^U]$ *and* $\tilde{B}^R = [\tilde{B}^L : \tilde{B}^U]$ be two trapezoidal fuzzy rough numbers and then greater-then and less-than operations can be defined as follows:
$$\tilde{A}^R \geq \tilde{B}^R \Leftrightarrow \tilde{A}^L \geq \tilde{B}^L \text{ and } \tilde{A}^U \geq \tilde{B}^U$$
$$\tilde{A}^R \leq \tilde{B}^R \Leftrightarrow \tilde{A}^L \leq \tilde{B}^L \text{ and } \tilde{A}^U \leq \tilde{B}^U$$
Also, we say $\tilde{A}^R = \tilde{B}^R \Leftrightarrow \tilde{A}^L = \tilde{B}^L$ *and* $\tilde{A}^U = \tilde{B}^U$.

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3. Problems formulation

3.1. Integer transportation problem with fully rough intervals

The general formula of the integer transportation problems with fully rough interval (ITPFRI) can be presented as:

$$(\textbf{ITPFRI})\quad Min\ Z^R = \sum_{i=1}^{m}\sum_{j=1}^{n}\left[\left[c_{ij}^{LL}, c_{ij}^{UL}\right]:\left[c_{ij}^{LU}, c_{ij}^{UU}\right]\right] \otimes \left[\left[x_{ij}^{LL}, x_{ij}^{UL}\right]:\left[x_{ij}^{LU}, x_{ij}^{UU}\right]\right]$$

$$\sum_{j=1}^{n}\left[\left[x_{ij}^{LL}, x_{ij}^{UL}\right]:\left[x_{ij}^{LU}, x_{ij}^{UU}\right]\right] = \left[\left[a_i^{LL}, a_i^{UL}\right]:\left[a_i^{LU}, a_i^{UU}\right]\right] \qquad (1)$$

$$\sum_{i=1}^{m}\left[\left[x_j^{LL}, x_j^{UL}\right]:\left[x_j^{LU}, x_j^{UU}\right]\right] = \left[\left[b_j^{LL}, b_j^{UL}\right]:\left[b_j^{LU}, b_j^{UU}\right]\right] \qquad (2)$$

$$x_{ij}^{LU},, x_{ij}^{LL}, x_{ij}^{UL} and\ x_{ij}^{UU} \geq 0 \qquad (3)$$
$$and\ \ integers\ variables$$

where $b_j^R = \left[\left[b_j^{LL}, b_j^{UL}\right]:\left[b_j^{LU}, b_j^{UU}\right]\right]$, $x_{ij}^R = \left[\left[x_{ij}^{LL}, x_{ij}^{UL}\right]:\left[x_{ij}^{LU}, x_{ij}^{UU}\right]\right]$, $c_{ij}^R = \left[\left[c_{ij}^{LL}, c_{ij}^{UL}\right]:\left[c_{ij}^{LU}, c_{ij}^{UU}\right]\right]$, $a_i^R = \left[\left[a_i^{LL}, a_i^{UL}\right]:\left[a_i^{LU}, a_i^{UU}\right]\right]$ and $z^R = \left[\left[z^{LL}, z^{UL}\right], \left[z^{LU}, z^{UU}\right]\right]$ $(i \in I, j \in J, I = 1, \ldots, m; J = 1, \ldots, n)$

m is the number of supply points; n is the number of demand points; x_{ij}^R is the rough intervals (RIs) of units shipped from supply point i to demand point j where $x_{ij}^{LU} \leq x_{ij}^{LL} \leq x_{ij}^{UL} \leq x_{ij}^{UU}$: c_{ij}^R is the rough cost of shipping one unit from supply point i to the demand point j; a_{ij}^R is the rough supply at supply point I ; b_{ij}^R is the rough interval demand at demand point j.
In the (ITPFRI) problem if the total supply is equal to the total demand, the (ITPFRI) problem is said to be balanced.

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Example 1: Consider the following fully rough integer TP:

	A	B	C	Supply
1	[(8,12):(7,14)]	[(12,17):(11,19)]	[(10,12):(9,16)]	[(14,16):(11,20)]
2	[(4,7):(2,10)]	[(5,8):(4,11)]	[(7,10):(6,9,13)]	[(12,14):(11,18)]
3	[(5,7):(3,11)]	[(3,5):(2,8)]	[(11,13):(10,16)]	[(15,17):(14,22)]
Demand	[(21,23):(18,28)]	[(12,14):(11,18)]	[(8,10):(7,14)]	[(41,47):(36,60)]

Now, since the total supply = the total demand = [[41,47]: [36,60]]the given problem is balanced.
Now, by using the proposed level-bound method we can get the optimal solution of the (ITPFRI) problem is given below

$\left[[x_{11}^{*LL}, x_{11}^{*UL}]: [x_{11}^{*LU}, x_{11}^{*UU}]\right]$ = [[6,6]: [4,6]],
$\left[[x_{13}^{*LL}, x_{13}^{*UL}]: [x_{13}^{*LU}, x_{13}^{*UU}]\right]$ = [[8,10]: [7,14]],
$\left[[x_{21}^{*LL}, x_{21}^{*UL}]: [x_{21}^{*LU}, x_{21}^{*UU}]\right]$ = [[12,14]:[11,18]],
$\left[[x_{31}^{*LL}, x_{31}^{*UL}]: [x_{31}^{*LU}, x_{31}^{*UU}]\right]$ = [[3,3]: [3,4]] and
$\left[[x_{32}^{*LL}, x_{32}^{*UL}]: [x_{32}^{*LU}, x_{32}^{*UU}]\right]$ = [[12,14]: [11,18]]
with the minimum shipping cost
$\left[[z_2^{*LL}, z_3^{*UL}]: [z_1^{*LU}, z_4^{*UU}]\right]$ = [[227,381]:[184,676]].

3.2. Integer transportation problems with fully trapezoidal fuzzy rough number

The general formula of the integer transportation problems with fully fuzzy rough trapezoidal number (ITPFFRTrN) may be presented as:

$$(\boldsymbol{ITPFFRTrN})\quad Min\ \left[\left[z_2^{LL}, z_3^M, z_4^N, z_5^{UL}\right] : \left[z_1^{LU}, z_3^M, z_4^N, z_6^{UU}\right]\right]$$

$$= \sum_{j=1}^{n} \left[\left[c_j^{LL}, c_j^M, c_j^N, c_j^{UL}\right] : \left[c_j^{LU}, c_j^M, c_j^N, c_j^{UU}\right]\right] \otimes \left[\left[x_{ij}^{LL}, x_{ij}^M, x_{ij}^N, x_{ij}^{UL}\right] : \left[x_{ij}^{LU}, x_{ij}^M, x_{ij}^N, x_{ij}^{UU}\right]\right]$$

$$\sum_{j=1}^{n} \left[\left[x_{ij}^{LL}, x_{ij}^M, x_{ij}^N, x_{ij}^{UL}\right] : \left[x_{ij}^{LU}, x_{ij}^M, x_{ij}^N, x_{ij}^{UU}\right]\right] = \left[\left[a_i^{LL}, a_i^M, a_i^N, a_i^{UL}\right] : \left[a_i^{LU}, a_i^M, a_i^N, a_i^{UU}\right]\right] \quad (4)$$

$$\sum_{j=1}^{n} \left[\left[x_{ij}^{LL}, x_{ij}^M, x_{ij}^N, x_{ij}^{UL}\right] : \left[x_{ij}^{LU}, x_{ij}^M, x_{ij}^N, x_{ij}^{UU}\right]\right] = \left[\left[b_j^{LL}, b_j^M, b_j^N, b_j^{UL}\right] : \left[b_j^{LU}, b_j^M, b_j^N, b_j^{UU}\right]\right] \quad (5)$$

$$x_{ij}^{LU}, x_{ij}^{LL}, x_{ij}^M, x_{ij}^N x_{ij}^{UL} and\ x_{ij}^{UU} \geq 0$$
$$and\ (Tr.F.N)\ integer\ variables \quad (6)$$

where $\left[\left[b_j^{LL}, b_j^M, b_j^N, b_j^{UL}\right] : \left[b_j^{LU}, b_j^M, b_j^N, b_j^{UU}\right]\right]$, $\left[\left[x_{ij}^{LL}, x_{ij}^M, x_{ij}^N, x_{ij}^{UL}\right] : \left[x_{ij}^{LU}, x_{ij}^M, x_{ij}^N, x_{ij}^{UU}\right]\right]$ $\left[\left[c_j^{LL}, c_j^M, c_j^N, c_j^{UL}\right] : \left[c_j^{LU}, c_j^M, c_j^N, c_j^{UU}\right]\right]$ and $\left[\left[a_i^{LL}, a_i^M, a_i^N, a_i^{UL}\right] : \left[a_i^{LU}, a_i^M, a_i^N, a_i^{UU}\right]\right]$ $(i \in I, j \in J, I = 1, \dots \dots, m; J = 1, \dots \dots, n)$ are positive integers and trapezoidal fuzzy rough number coefficients and variables of the objective function and the constraints also, to be balanced if the total supply is equal to the total demand. Where $x_{ij}^{LU} \leq x_{ij}^{LL} \leq x_{ij}^M \leq x_{ij}^N \leq x_{ij}^{UL} \leq x_{ij}^{UU}$.

Definition 6. A set of rough intervals $\left\{\left[\left[x_{ij}^{LL}, x_{ij}^M, x_{ij}^N, x_{ij}^{UL}\right] : \left[x_{ij}^{LU}, x_{ij}^M, x_{ij}^N, x_{ij}^{UU}\right]\right] \text{, for all } i \in I\ and\ j \in J\right\}$ } is said to be a feasible to the **(**ITPFFRTrN**)** problem if it satisfies the equations (4), (5) and (6).

Definition 7. A feasible solution $\left\{\left[\left[x_{ij}^{LL}, x_{ij}^M, x_{ij}^N, x_{ij}^{UL}\right] : \left[x_{ij}^{LU}, x_{ij}^M, x_{ij}^N, x_{ij}^{UU}\right]\right] \text{, for all } i \in I\ and\ j \in J\right\}$ to the **(**ITPFFRTrN**)** problem is said to be an optimal solution of the problem **(**ITPFFRTrN**)** if the feasible solution minimizes the objective function of the **(**ITPFFRTrN**)** , that is

$$\sum_{i=1}^{m} \sum_{j=1}^{n} \left[\left[c_j^{LL}, c_j^M, c_j^N, c_j^{UL}\right] : \left[c_j^{LU}, c_j^M, c_j^N, c_j^{UU}\right]\right]$$
$$\otimes \left[\left[x_{ij}^{LL}, x_{ij}^M, x_{ij}^N, x_{ij}^{UL}\right] : \left[x_{ij}^{LU}, x_{ij}^M, x_{ij}^N, x_{ij}^{UU}\right]\right] \leq$$

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$$\sum_{i=1}^{m}\sum_{j=1}^{n}\left[\left[c_j^{LL},c_j^{M},c_j^{N},c_j^{UL}\right]:\left[c_j^{LU},c_j^{M},c_j^{N},c_j^{UU}\right]\right]$$
$$\otimes\left[\left[x_{ij}^{LL},x_{ij}^{M},x_{ij}^{N},x_{ij}^{UL}\right]:\left[x_{ij}^{LU},x_{ij}^{M},x_{ij}^{N},x_{ij}^{UU}\right]\right]$$

for all feasible $\left\{\left[\left[x_{ij}^{LL},x_{ij}^{M},x_{ij}^{N},x_{ij}^{UL}\right]:\left[x_{ij}^{LU},x_{ij}^{M},x_{ij}^{N},x_{ij}^{UU}\right]\right], \text{for all } i\in I \text{ and } j\in J\right\}$.

Now, the (ITPFFRTrN) problem is sliced into 6 integer transportation problems namely, 1st level integer TP **(**ITPFFRTrN1), 2nd level integer TP **(**ITPFFRTrN2), 3rd level integer TP **(**ITPFFRTrN3**)**, 4th level integer TP **(**ITPFFRTrN4), 5th level integer TP **(**ITPFFRTrN5**)**, 6th level integer TP **(**ITPFFRTrN6), as given below:

$$(ITPFFRTrN1)\quad Min\ \ Z_1^{LU}=\sum_{i=1}^{m}\sum_{j=1}^{n}c_{ij}^{LU}\,x_{ij}^{LU}$$

$$Subject\ to\quad \sum_{j=1}^{n}c_{ij}^{LU},=a_{ij}^{LU}\ \ ,i\in I\,;\sum_{j=1}^{n}c_{ij}^{LU}=b_j^{LU}\ \ ,j\in J:$$
$$x_{ij}^{LU}\geq 0,\ \ i\geq I\ \ \ and\ j\geq J\ are\ integers.$$

$$(ITPFFRTrN2)\quad Min\ \ Z_2^{LL}=\sum_{i=1}^{m}\sum_{j=1}^{n}c_{ij}^{LL}\,x_{ij}^{LL}$$

$$Subject\ to\quad \sum_{j=1}^{n}c_{ij}^{LL},=a_{ij}^{LL}\ \ ,i\in I\,;\sum_{j=1}^{n}c_{ij}^{LL}=b_j^{LL}\ \ ,j\in J:$$
$$x_{ij}^{LL}\geq 0,\ \ i\geq I\ \ \ and\ j\geq J\ are\ integers.$$

$$(ITPFFRTrN3)\quad Min\ \ Z_3^{M}=\sum_{i=1}^{m}\sum_{j=1}^{n}c_{ij}^{M}\,x_{ij}^{M}$$

$$Subject\ to\quad \sum_{j=1}^{n}c_{ij}^{M},=a_{ij}^{M}\ \ ,i\in I\,;\sum_{j=1}^{n}c_{ij}^{M}=b_j^{M}\ \ ,j\in J:$$
$$x_{ij}^{M}\geq 0,\ \ i\geq I\ \ \ and\ j\geq J\ are\ integers.$$

$$(ITPFFRTrN4)\quad Min\ \ Z_4^{N}=\sum_{i=1}^{m}\sum_{j=1}^{n}c_{ij}^{N}\,x_{ij}^{N}$$

$$\text{Subject to}\quad \sum_{j=1}^{n}c_{ij}^{N},=a_{ij}^{N}\ \ ,i\in I\,;\sum_{j=1}^{n}c_{ij}^{N}=b_j^{N}\ \ ,j\in J:$$

$$x_{ij}^{N} \geq 0, \quad i \geq I \quad and\ j \geq J\ are\ integers.$$

$$(ITPFFRTrN5) \quad Min \quad Z_5^{UL} = \sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij}^{UL}\, x_{ij}^{UL}$$

$$\text{Subject to} \quad \sum_{j=1}^{n} c_{ij}^{UL}, = a_{ij}^{UL} \quad , i \in I\,; \sum_{j=1}^{n} c_{ij}^{\mathrm{UL}} = b_j^{UL} \quad , j \in J:$$

$$x_{ij}^{UL} \geq 0, \quad i \geq I \quad and\ j \geq J\ are\ integers.$$

$$(ITPFFRTrN6) \quad Min \quad Z_6^{UU} = \sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij}^{UU}\, x_{ij}^{UU}$$

$$Subject\ to \quad \sum_{j=1}^{n} c_{ij}^{UU}, = a_{ij}^{UU} \quad , i \in I\,; \sum_{j=1}^{n} c_{ij}^{UU} = b_j^{UU} \quad , j \in J:$$

$$x_{ij}^{UU} \geq 0, \quad i \geq I \quad and\ j \geq J\ are\ integers.$$

Where the rough value optimal solutions $\tilde{Z}^R$ and decision rough integer variables $\tilde{x}_j^R$ in problem **(**ITPFFRTrN**)** will be as:

$$\tilde{Z}^R = \left[[z_2^{LL}, z_3^{M}, z_4^{N}, z_5^{UL}] : [z_1^{LU}, z_3^{M}, z_4^{N}, z_6^{UU}] \right]$$

$$\tilde{x}_{ij}^R = \left[[\, x_{ij}^{LL}, x_{ij}^{M}, x_{ij}^{N}, x_{ij}^{UL}] : [x_{ij}^{LU}, x_{ij}^{M}, x_{ij}^{N}, x_{ij}^{UU}] \right]$$

$$(\, i \in I, j \in J, I = 1, \dots \dots, m; J = 1, \dots \dots, n\,)$$

3.3. Solution procedures for ITPFFRTrN

The slides solution mothed proceeds as follows :

Step 1: Check that the given (ITPFFRTrN**)** problem is balanced. If not, make it into balanced.

Step 2: Construct 6th level TP problems of the given (ITPFFRTrN**)** problem.

Step 3: Solve the 6th level TP problem using a transportation algorithm. Let $\{x_{ij}^{*UU}$, for all $i \in I\ and\ j \in J\}$ be an optimal solution of the 6th level TP problem with the minimum transportation cost z_6^{*UU}.

Step 4 : Solve the 5th level TP problem with the upper bound constraints $x_{ij}^{UL} \leq x_{ij}^{*UU}$ for all $i \in I\ and\ j \in J$ using Vogel method and the integer linear programming technique. Let $\{x_{ij}^{*UL}$, for all $i \in I\ and\ j \in$

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$J\}$ be an optimal solution of the 5th level TP problem with the minimum transportation cost z_5^{*UL}.

Step 5 : Solve the 4th level TP problem with the upper bound constraints $x_{ij}^{N} \leq x_{ij}^{*UL}$ for all $i \in I\ and\ j \in J$ using Vogel method and the integer linear programming technique. Let $\{x_{ij}^{*N}$, for all $i \in I\ and\ j \in J\}$ be an optimal solution of the 4th level TP problem with the minimum transportation cost z_4^{*N}.

Step 6 : Solve the 3rd level TP problem with the upper bound constraints $x_{ij}^{M} \leq x_{ij}^{*N}$ for all $i \in I\ and\ j \in J$ using Vogel method and the integer linear programming technique. Let $\{x_{ij}^{*M}$, for all $i \in I\ and\ j \in J\}$ be an optimal solution of the 3rd level TP problem with the minimum transportation cost z_3^{*M}.

Step 7 : Solve the 2nd level TP problem with the upper bound constraints $x_{ij}^{LL} \leq x_{ij}^{*M}$, for all $i \in I\ and\ j \in J$ using Vogel method and the integer linear programming technique. Let $\{x_{ij}^{*LL}$, for all $i \in I\ and\ j \in J\}$ be an optimal solution of the 2nd level TP problem with the minimum transportation cost z_2^{*LL}.

Step 8 : Solve the 1st level TP problem with the upper bound constraints $x_{ij}^{LU} \leq x_{ij}^{*LL}$ for all $i \in I\ and\ j \in J$ using Vogel method and the integer linear programming technique to solve. Let $\{x_{ij}^{*LU}$, for all $i \in I\ and\ j \in J\}$ be an optimal solution of the 1st level TP problem with the minimum transportation cost z_1^{*LU}.

Step 9 : The optimal solution of the given problem **(**ITPFFRTrN**)** is $\left[\left[\, x_{ij}^{LL}, x_{ij}^{M}, x_{ij}^{N}, x_{ij}^{UL}\right] : \left[x_{ij}^{LU}, x_{ij}^{M}, x_{ij}^{N}, x_{ij}^{UU}\right]\right]$ with the minimum transportation cost $\left[[z_2^{LL}, z_3^{M}, z_4^{N}, z_5^{UL}] : [z_1^{LU}, z_3^{M}, z_4^{N}, z_6^{UU}]\right]$.

The solution procedure of the proposed method for solving the fully fuzzy rough integer trapezoidal TP is illustrated by the following numerical example.

***Example* 2 :** Consider the following fully fuzzy rough integer trapezoidal TP:

	A	B	C	Supply
1	[(6, 7,9,10): (4,7, 9,13)]	[(11,12,14,15): (9,12, 14,18)]	[(8, 9,10,11): (6,9, 10, 13)]	[(12, 13,15,16): (10,13, 15,19)]
2	[(3,4,5,6): (1,4, 5,8)]	[(5,6,7,8): (2,6,7,10)]	[(6, 7,8,9): (3,7, 8,12)]	[(10, 11,13,14): (8,11, 13,17)]
3	[(4,6,7,8): (2,6,7,10)]	[(3,4,6,7): (1,4,6,9)]	[(9,10,11,12): (7,10, 11,14)]	[(13, 14,16,18): (11,14, 16,21)]
Demand	[(19,20,22,24): (17,20, 22,27)]	[(10, 11,13,14): (8,11, 13,17)]	[(6, 7,9,10): (4,7,9,13)]	[(35,38,44,48): (29,38,44,57)]

Now, since the total supply = the total demand = [[35,38,44,48], [29,38 ,44,57]] the given problem is balanced.
Now, by using the proposed level-bound method we can get the optimal solution of the problem (ITPFFRTrN**)** as follows:

$\left[\left[x_{11}^{*LL}, x_{11}^{*M}, x_{11}^{*N}, x_{11}^{*UL}\right]:\left[x_{11}^{*LU}, x_{11}^{*M}, x_{11}^{*N}, x_{11}^{*UU}\right]\right]$ = [[12,13,15,16]: [10,13,15,19]],

$\left[\left[x_{21}^{*LL}, x_{21}^{*M}, x_{21}^{*N}, x_{21}^{*UL}\right]:\left[x_{21}^{*LU}, x_{21}^{M}, x_{21}^{*N}, x_{21}^{*UU}\right]\right]$ = [[7,7,7,8]: [7,7,7,8]],

$\left[\left[x_{23}^{*LL}, x_{23}^{*M}, x_{23}^{*N}, x_{ij}^{*UL}\right]:\left[x_{23}^{*LU}, x_{23}^{*M}, x_{23}^{*N}, x_{23}^{*UU}\right]\right]$ = [[3,4,6,6]: [1,4,6,8]],

$\left[\left[x_{32}^{*LL}, x_{32}^{*M}, x_{32}^{*N}, x_{32}^{*UL}\right]:\left[x_{32}^{*LU}, x_{32}^{*M}, x_{32}^{*N}, x_{32}^{*UU}\right]\right]$ = [[10,11,13,14]: [8,11,13,17]] and

$\left[\left[x_{33}^{*LL}, x_{33}^{*M}, x_{33}^{*N}, x_{33}^{*UL}\right]:\left[x_{33}^{*LU}, x_{33}^{*M}, x_{33}^{*N}, x_{33}^{*UU}\right]\right]$ = [[3,3,3,4]: [3,3,3,4]]

with the minimum shipping cost

$\tilde{Z}^{*R} = \left[\left[z_2^{LL}, z_3^{M}, z_4^{N}, z_5^{UL}\right]:\left[z_1^{LU}, z_3^{M}, z_4^{N}, z_6^{UU}\right]\right]$ = [[168,221,329, 408]: [79,221,329,628]]

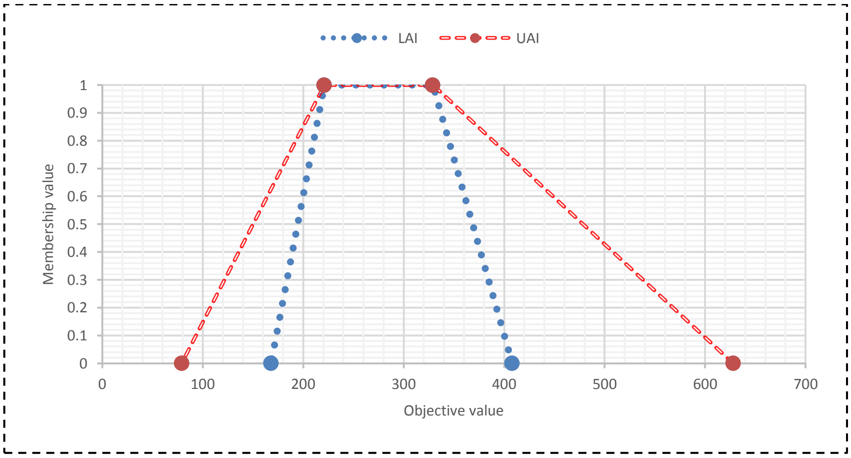

Fig (2) $\tilde{Z}^{*R}$ trapezoidal fuzzy rough objective values for example 3

4 Discussion

Compared with both the linear programming with rough Interval coefficients by (Hamzehee et al. 2014) who only used the rough interval in linear programming problems, and with fuzzy interval integer transportation problems by (Pandian et al. 2016) who used a new method namely; level-bound method that was proposed to solve fuzzy interval integers transportation problems, this study used trapezoidal fuzzy rough number coefficients and variables of the objective function and the constraints to find an optimal solution and optimal values cost where the supply and demand are balanced. The focus of our study is to develop an improved method to solve integer transportation problems, where all parameters and decision variables in the constraints and the objective function are trapezoidal fuzzy rough number. Integer programming is used, since many transportation problems in our real life require that the decision variables be integers. In addition, fuzzy number rough intervals are very important to tackle the uncertainty and imprecise data in decision making problems. Moreover, the proposed algorithm enables us to search for the optimal solution in the largest range of possible solutions. The motivation behind the study is to enable the decision maker to make the right decision in the field of proposed solutions, in case of having to deal with the uncertainty and imprecise data.

5 Conclusion

The cost of transportation from the source to destination is considered to be rough costs are assigned. The availability as well as the demand is also considered to be rough interval parameters. This work deals with two

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application problems the first one is for solve fully rough interval integer transportation problems, the second is all parameters and decision variables in the constraints and the objective function are trapezoidal fuzzy rough number TrFRN. For all above two application problems are obtained and discussed optimal solution and optimal values cost. by slides solution method. This method is systematic procedure, both easy to understand and to apply. The proposed method provides more options and can be served an important tool for the decision makers when they are handling various types of logistic problems having rough interval parameters. We think the slides solution mothed is useful as new tools to tackle the uncertainty, vague and imprecise data in decision making problems.

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